Discontinuities in Symmetric Striplines Due to Impedance Steps and Their Compensations

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Abstract—The theoretically known magnitude of the series lumped reactance, as a function of impedance step ratio, resulting from an impedance step discontinuity in symmetric stripline is confirmed, and an alternative expression for the inductance is given.

A reduction of this reactance has been achieved by splitting the narrower strip at the impedance step junction into a multistrip configuration, while retaining the total characteristic impedance value.

A way of compensating the effect of the reactance on the passband characteristics of quarter-wave impedance transformers and filters is developed. This is achieved by the introduction of a lumped series capacitance at the impedance step discontinuity.

I. Introduction

In 1955, Oliner [1] indicated that when impedance step discontinuities occur in symmetric striplines,[1] reactances are introduced. An approximate formula for this reactance as a function of the impedance step ratio was given. Successful experimental measurements have not been made previously to verify Oliner's formula. Microstrip and stripline designers have, to date, overlooked this lumped reactance and its effect on the passband characteristics.

The purpose of this investigation was to experimentally confirm the existence of the series lumped reactance and its relationship to the impedance step ratios, to investigate the effect of the inductance on quarter-wave transformers, and to give methods to reduce or compensate for this effect.

II. The Equivalent Circuit for a Step Discontinuity

The approximate magnetic field lines (found experimentally by field mapping) of a stripline with step discontinuity in the center conductor (top view) are similar to the electric field lines of a parallel-plate transmission line with step discontinuity in the plate separation, as seen in Figs. 1 and 2. The normalized equivalent circuit for a step discontinuity of the plate spacing of a parallel-plate transmission line has been given by several authors [1]–[4]. The most detailed and accurate derivation has

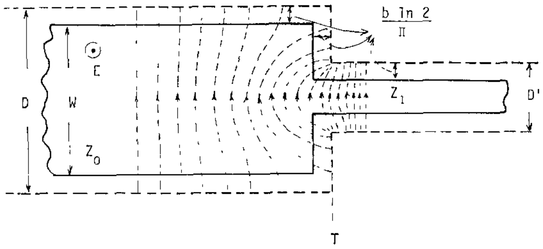

Fig. 1. The magnetic field lines of a stripline near a step discontinuity (top view).

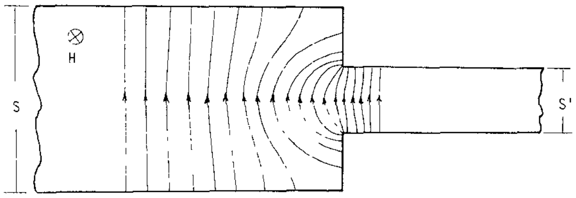

Fig. 2. The electric field lines of a parallel-plate transmission line near a step discontinuity (side view).

been given by Schwinger [4]. This discontinuity effectively is a shunt capacitance with a normalized susceptance

$$\frac{B}{Y_0} \simeq \frac{2S}{\lambda} K \qquad (1)$$

where

$$K = \ln\left[\left(\frac{1-\alpha^2}{4\alpha}\right)\left(\frac{1+\alpha}{1-\alpha}\right)^{(\alpha+1/\alpha)/2}\right] + \frac{2}{A}.$$

Subsequently,

$$\alpha = \frac{S'}{S} < 1 \qquad (2a)$$

and

$$A = \left(\frac{1+\alpha}{1-\alpha}\right)^{2\alpha}\left[\frac{1+(1-S^2/\lambda^2)^{1/2}}{1-(1-S^2/\lambda^2)^{1/2}}\right] - \frac{1+3\alpha^2}{1-\alpha^2}. \qquad (2b)$$

In (1) and (2), λ is the wavelength in a medium of dielectric constant ϵ_r and both S' and S are the narrow and the wide spacings between the plates, respectively (Fig. 2). By applying the duality principle [4, chapter 3, section 10] to (1), the equivalent circuit (Fig. 3) for

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[1] Hereafter, "stripline" will be understood to refer to symmetric stripline.

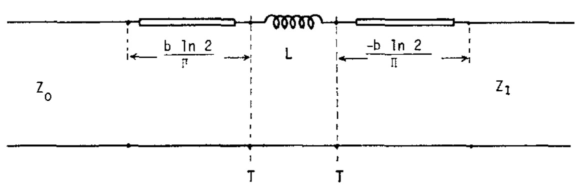

Fig. 3. The equivalent circuit of Fig. 1.

a step discontinuity in the center strip width (Fig. 1) includes a series inductor. The inductive reactance for the case of zero thickness center strip is

$$\frac{X}{Z_0} \simeq \frac{2DK}{\lambda} \tag{3}$$

where A and α can be found when S' and S in (1) and (2) are replaced by D' and D, respectively. D' and D are the equivalent strip widths; an accurate value of D is given by [5] and, to a good approximation, for $w/b > 0.5$, by

$$D = \frac{30\pi b}{Z_0\sqrt{\epsilon_r}} \simeq w + \frac{2b}{\pi}\ln 2 \tag{4}$$

where b is the spacing between the ground plates and w is the physical center strip width. Z_0 in (3) corresponds to the characteristic impedance of the stripline of width w.

Equation (3) can be rewritten as

$$L = \frac{30b}{c}K \tag{5}$$

where c is the velocity of light in free space. The lumped inductor and the impedance step are at a distance of very nearly $(b \ln 2)/\pi$ away from the physical junction because of the fringing field lines [5], as represented by the line lengths shown in Fig. 3.

Oliner [1], [5], also using the duality principle, obtained a series inductance

$$L = \frac{30b}{c}\ln\left[\csc\left(\frac{\pi}{2}\alpha\right)\right]. \tag{6}$$

This result applies for a $b/\lambda \cong 0$, and when α approaches zero, (6) approaches (5) as shown in Fig. 7.

III. The Effect of the Series Lumped Inductors on the Reflection Coefficient of Quarter-Wave Transformers

The transfer matrix for a single-section transformer of length d and normalized characteristics impedance of Z_1, with normalized lumped series reactances X_1 and X_2 at each end, is

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \cos\beta d \begin{bmatrix} 1 & jX_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & jZ_1 \tan\beta d \\ \dfrac{j\tan\beta d}{Z_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & jX_2 \\ 0 & 1 \end{bmatrix} \tag{7}$$

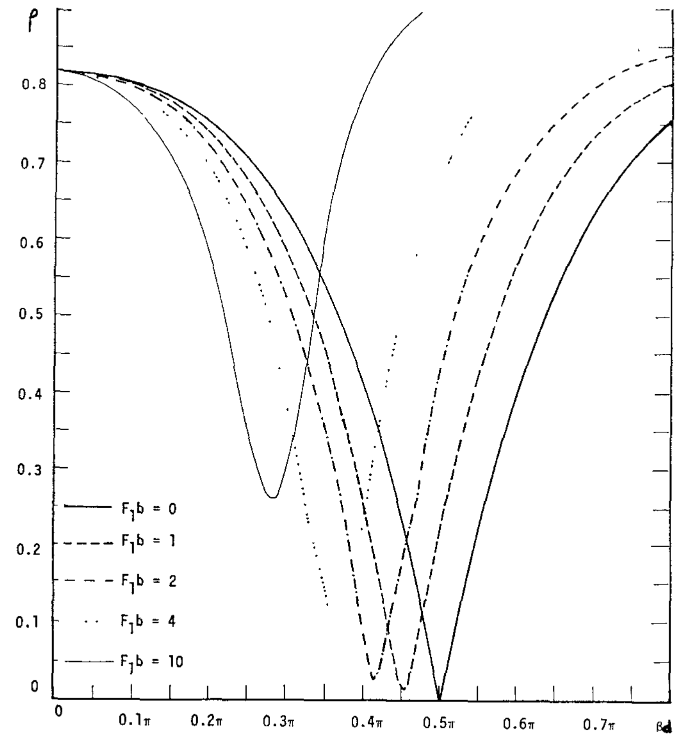

Fig. 4. The input reflection coefficient versus the frequency for a one-section transformer with inductors of different reactances at the impedance step discontinuities.

where β is the phase constant per unit length of the line.

Extending this to the transfer matrix for the two-section quarter-wave transformer with three lumped series reactances gives

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \cos^2\beta d \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & jZ_2 \tan\beta d \\ \dfrac{j\tan\beta d}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & jX_3 \\ 0 & 1 \end{bmatrix}. \tag{8}$$

The magnitude of the reflection coefficient of a transformer placed between impedances Z_0 and Z_L is [6]

$$|\rho(j\omega)| = \left[\frac{(A_iZ_L - D_iZ_0)^2 + (B_i - C_iZ_0Z_L)^2}{(A_iZ_L + D_iZ_0)^2 + (B_i + C_iZ_0Z_L)^2}\right]^{1/2}, \quad i = 1, 2. \tag{9}$$

The passband characteristics of one- and two-section (maximally flat) impedance transformers, matching impedances of 50 Ω (Z_0) and 5 Ω (Z_L), are given in Figs. 4 and 5, respectively, for various values of F_1b, where b is the stripline ground-plate separation in centimeters and F_1 corresponds to the frequency (in gigahertz) for which each line section is a half wavelength long. Figs. 4 and 5 show that as the series reactance increases (or F_1b increases), the passband narrows, shifts to lower frequencies, and the reflection coefficient of the passband increases. These effects may be minimized with the methods that will be outlined in Sections V and VI.

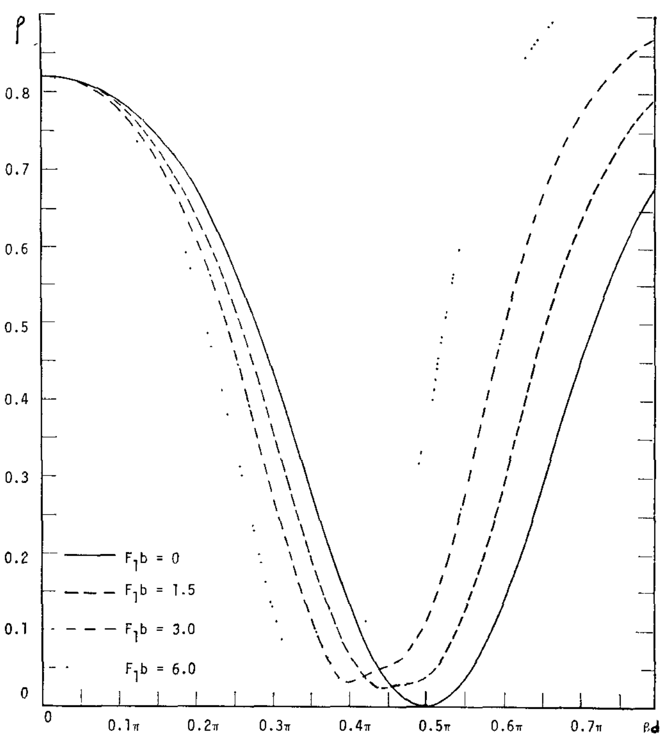

Fig. 5. The input reflection coefficient versus the frequency for a two-section maximally flat transformer with inductors of difference reactances at the impedance step discontinuities.

IV. The Experimental Determination of the Series Inductance of Step Impedance Discontinuities

The method developed for measuring the series inductance is based on the stripline configuration of Fig. 6(a). When the strip at the right is match terminated, the normalized input impedance to the center section of impedance Z_1 is

$$Z_i' = Z_1' \frac{(1 + jX') \cos \beta d + Z_1' j \sin \beta d}{Z_1' \cos \beta d + (1 + jX') j \sin \beta d} + jX'. \quad (10)$$

For zero reflection, $Z_i' = 1$ and (10) can be solved for X' to yield

$$X' = Z_1' \cot \beta d \pm (Z_1'^2 \csc^2 \beta d - 1)^{1/2}. \quad (11)$$

The positive sign in (11) is used when $Z_1' > 1$; the negative sign is used when $Z_1' < 1$. To find X' experimentally, the frequency is varied and whenever the reflection coefficient is a minimum, (11) is met. The reflection coefficient might not reach zero due to small reflections in the system. Knowing the values of Z_1', d, and $\beta = ((2\pi(\epsilon_r)^{1/2})/(\lambda_0))$, the values of the lumped inductances may be calculated.

Equation (11) also holds for the configuration of Fig. 6(b), but X and Z_1 should be normalized to the characteristic impedance of the wider strip (10 Ω) and the tapered impedance transformers should have very low reflection coefficients. The tapered impedance transformers that were used have a reflection coefficient $\rho < 0.07$ when $Z_1 = 10$ Ω.

The comparison of the experimental results for Fig. 6(a) or (b) with the theoretically derived values of the lumped inductances is shown in Fig. 7, where errors in Z_1' and frequency measurements were taken into consideration.

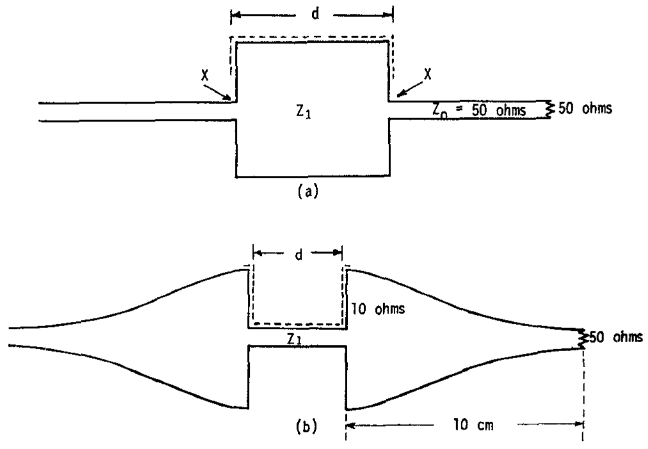

Fig. 6. The center strip configurations used to measure the series inductance.

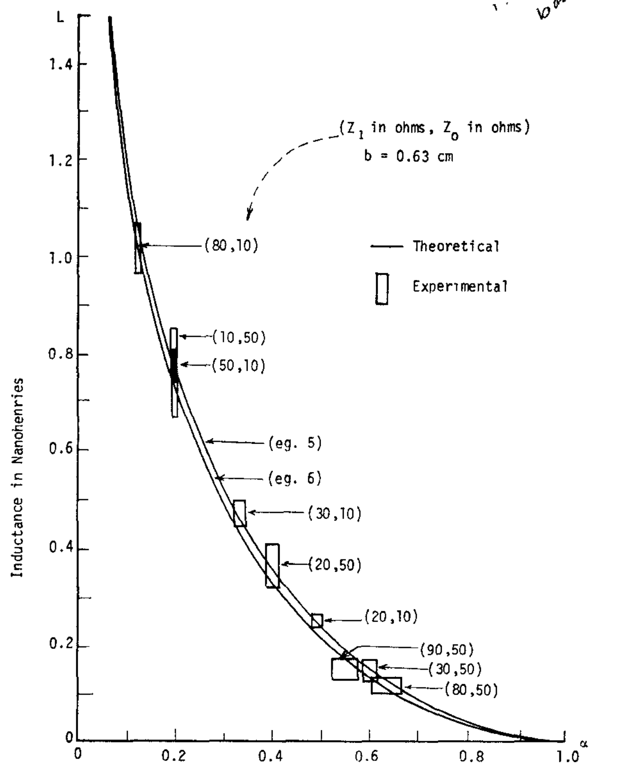

Fig. 7. The comparison of the theoretical and the experimental values of the discontinuity inductors as a function of impedance ratio.

V. The Reduction of the Lumped Inductors by Means of Multistrips

In a stripline circuit, one strip of higher impedance between two wider strips of lower impedance can be split into n multiple strips with the equivalent total characteristic impedances of the single strip. This will not affect the circuit characteristics, assuming junction lumped inductors do not exist. In this section, the

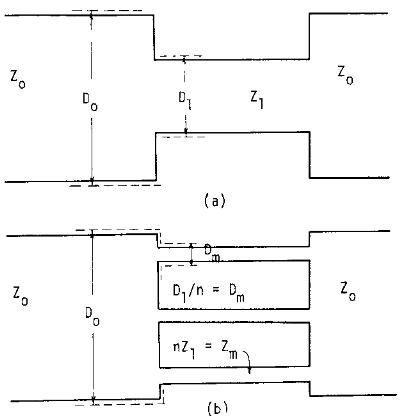

Fig. 8. A single strip and its equivalent multistrip circuit.

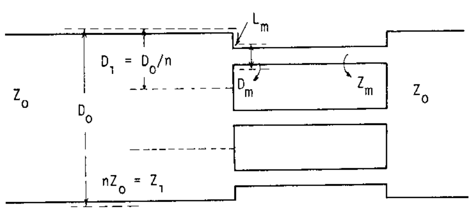

Fig. 9. The illustration of an intermediate transition portion between a wide strip and narrow multistrips.

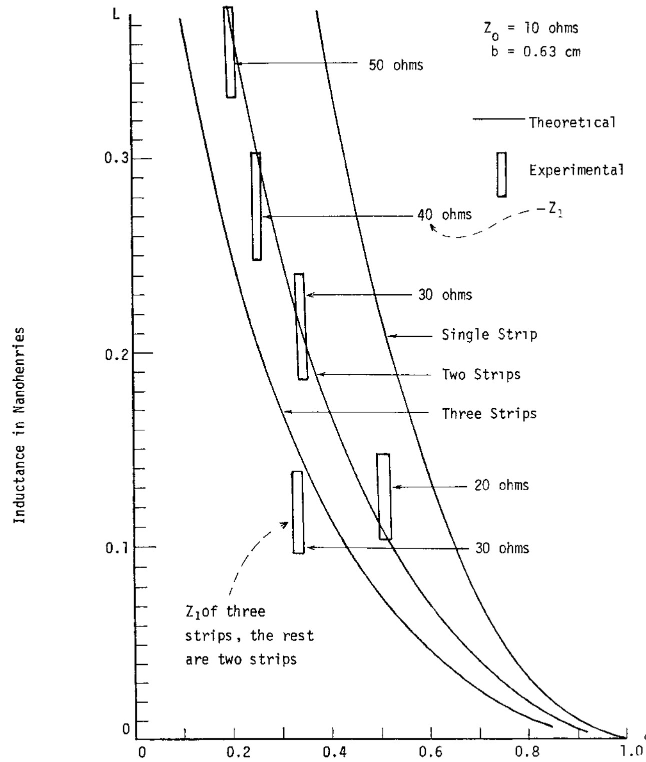

Fig. 10. The comparison of the theoretical and the experimental values of inductances of discontinuities involving multistrip configuration.

change in junction lumped inductance due to this arrangement will be investigated. The characteristic impedances and the effective width of these striplines are illustrated in Fig. 8.

Consider the wider strip to be divided into n parts of equal effective widths D_i ($D_i = D_0/n$) by an imaginary dotted line just before it is split into n narrower multistrips, as seen in Fig. 9. Let this stage be called the intermediate stage.

Since the current going from the wide strip into the intermediate stage does not experience any path change, there will not be any junction lumped reactances. Also, the total characteristic impedance does not change. Carrying equal currents, the strips are also at equal voltages to ground, and are therefore uncoupled.

Let the characteristic impedance and the effective width of each of the narrow multistrips be Z_m and D_m, respectively. Let L_m be the junction lumped inductance between one narrow strip (of effective width D_m) and one part of the intermediate stage (of effective width D_i). This inductance can be obtained from (6):

$$L_m = \frac{30b}{c} \ln \csc\left(\frac{\pi}{2}\frac{D_m}{D_i}\right) = \frac{30b}{c} \ln \csc\left(\frac{\pi}{2}\frac{Z_i}{Z_m}\right). \quad (12)$$

Let L_1 be the junction inductance between the wide strip (of effective width D_0) and the single strip (of effective width D_1). Then

$$L_1 = \frac{30b}{c} \ln \csc\left(\frac{\pi}{2}\frac{D_1}{D_0}\right) = \frac{30b}{c} \ln \csc\left(\frac{\pi}{2}\frac{Z_0}{Z_1}\right) \quad (13)$$

since

$$\frac{D_1}{D_0} = \frac{D_m}{D_i} = \frac{Z_0}{Z_1} = \frac{Z_i}{Z_m}. \quad (14)$$

It is found by substitution in (13) and (14) that

$$L_1 = L_m. \quad (15)$$

Therefore, the lumped inductance (L_1) between the wide strip and the single strip [Fig. 8(a)] is equal to the lumped inductance (L_m) between an intermediate stage of the wide strip and a multistrip. L_T is the equivalent of n inductances L_m in parallel:

$$L_T = \frac{L_1}{n} = \frac{L_m}{n}. \quad (16)$$

The experimental procedure of measuring the lumped inductance at the junction of a single strip and the multistrip parallel lines is the same as the procedure used to measure the lumped inductance at the junction of two single strips. The results of the measurements for two and three parallel multistrips are shown in Fig. 10 and are compared with the theoretical values.

VI. The Compensation of the Inductor by a Lumped Capacitor

The effect of the series inductor at an impedance step discontinuity may be reduced by placing a lumped capacitor in series with the inductor so that the total reactance is zero at a given frequency (e.g., at band center).

A lumped junction capacitor can be realized by

TABLE I
THE COMPARISON OF THE EXPERIMENTAL AND THE THEORETICAL VALUES OF THE LUMPED CAPACITANCES BETWEEN TWO OVERLAPPING CENTER CONDUCTING STRIPS

Dielectric Thickness (mm)	Frequency of VSWR Minimum (GHz)	Total Reactance (Ω)	Capacitance (pF) (Experimental)	Capacitance (pF) (Theoretical)
0.000	3.151±0.008	15.10±0.20	∞	∞
0.025	3.312±0.010	12.20±0.20	13.03±1.30	8.9±1.2
0.050	3.435±0.010	9.71±0.20	6.84±0.56	4.5±0.6
0.075	3.532±0.010	7.79±0.20	4.91±0.30	3.0±0.4
0.100	3.628±0.010	5.90±0.20	3.81±1.22	2.22±0.3

separating the center conductors at the junction by means of a gap. If s is the gap width, the equivalent circuit will consist of a lumped series capacitor with a capacitance of [1]

$$C = \frac{b(\epsilon_r)^{1/2}}{2\pi c Z_0} \ln \left[\coth \left(\frac{\pi s}{2b} \right) \right] \tag{17}$$

and two shunt inductors which are ignored because their susceptance approaches zero as $s/b<0.1$, which is the condition used to get a capacitor as large as possible. The capacitance obtained by this method is generally too small to compensate the lumped series capacitance.

Another method of introducing a lumped series capacitor large enough to compensate the inductor is by overlapping the center strips with a very thin dielectric film between. The length of the overlap should be small (less than 0.05λ) in order to consider the capacitors as lumped.

The concept of compensating a stripline step junction lumped inductance by the introduction of a lumped capacitance was tested experimentally. The configuration shown in Fig. 6(b) was modified by making the 10-Ω stripline overlap the tapered line sections by 2.0 mm at each end. One or more layers of 0.025-mm thick Mylar sheet pieces were placed between the overlap, forming lumped capacitors. The total reactance at the junction was measured (see Table I) for zero to four layers of Mylar sheet. Knowing the inductive reactance from the first measurement, the capacitive reactance was calculated, and from this the capacitance value was obtained. Comparison with the theoretical (parallel-plate, without fringing) capacitance shows some deviation which can be attributed to the fringing capacitance and the assumption of constant inductance with decreasing capacitance values.

The inaccuracies of the measurement system are reflected in the second and third columns of Table I, the resulting inaccuracy in measured capacitance being around ±10 percent. The error resulting from inaccuracy in the physical dimensions of the overlapping conductors further accounts for the discrepancies between measured and computed capacitance.

The effect of capacitive compensation of series lumped inductors on the passband characteristics of a one-section quarter-wave impedance transformer is shown in Fig. 11 for several different values of capacitors. When the length of the transformer is a quarter-wave long, the reflection coefficient at the center of the passband is zero, but the passband is narrower.

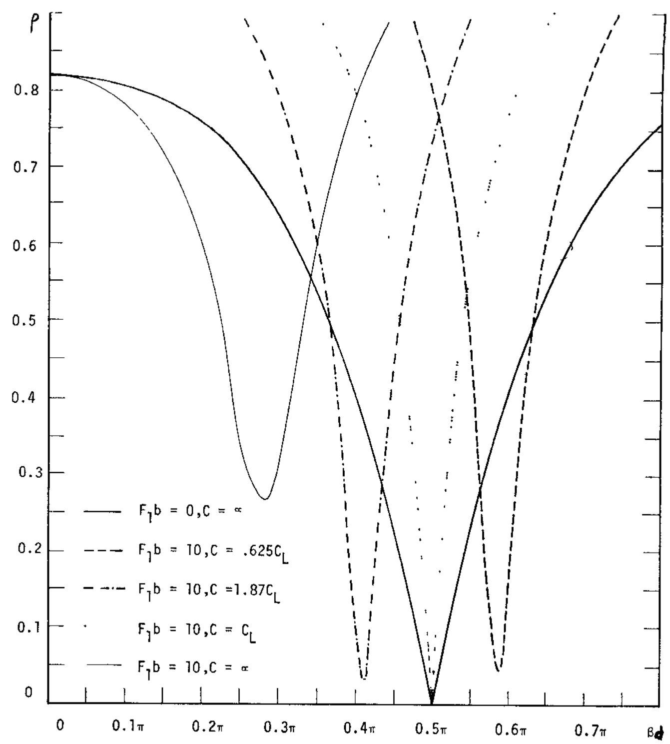

Fig. 11. The input reflection coefficient versus the frequency for a one-section transformer with capacitive compensation of the inductors.

VII. CONCLUSION

An alternative formulation of the magnitude of the series lumped inductance in the equivalent circuit of stripline impedance step discontinuity has been obtained. This inductance has been measured and the results have been compared to the theoretical values. It was found that the theoretical values computed from the two formulations were in substantial agreement with each other and with measurements.

A means of reducing this series lumped inductance has been developed by splitting the narrower strip at the impedance step junction into multistrips while retaining the characteristic impedance value.

It was found that this series inductance narrows the passband of impedance transformers, shifts the passband to lower frequencies, and increases the passband reflection coefficient.

Introduction of a lumped series capacitor at the impedance step discontinuity successfully compensated for this inductor, made the passband of the impedance transformer resemble the ideal one, and gave the stripline circuit designer a means of reducing the effect of this inductance.

Due to similarities between stripline and microstrips, all the results presented should be qualitatively applicable to microstrips also.

Further work will be required to fully evaluate the effect of the inductive reactance, together with its compensations, in microstrip and stripline designs of impedance transformers and filters.

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Waveguides of Arbitrary Cross Section by Solution of a Nonlinear Integral Eigenvalue Equation

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Abstract—**The problem of electromagnetic wave propagation in hollow conducting waveguides of arbitrary cross section is formulated as an integro-differential equation in terms of fields at the waveguide boundary. Cutoff wave numbers and wall currents appear as eigenvalues and eigenfunctions of a nonlinear eigenvalue problem involving an integro-differential operator. A variational solution is effected by reducing the problem to matrix form using the method of moments.**

A specific solution of the problem is developed using triangle expansion functions in the method of moments. The solution is simplified by symmetry considerations and is implemented by two digital computer programs. Listings and full documentation of these programs are available. This solution yields accurate determinations of cutoff wave numbers, wall currents, and distributions of both longitudinal and transverse modal field components for the first several modes. Illustrative computations are presented for the single-ridge waveguide, which has a complicated boundary shape that does not lend itself to exact solution.

I. Introduction

ELECTROMAGNETIC wave propagation in hollow conducting waveguides of arbitrary cross section is a problem of considerable interest. An interesting review paper by Davies [1] gives a comparative discussion of many of the methods previously applied to this general problem. His discussion makes clear that no single solution method has proved to be best for all requirements that might be imposed.

In this paper a new solution for waveguides of arbitrary cross section is presented. The approach is based on an integral operator formulation which affords a unified treatment of the various classes of waveguide shape. In principle, the first several modes can be analyzed provided the boundary of the waveguide is closed. The convergence characteristics and accuracy of the method have been demonstrated previously [2]. Example calculations of cutoff wave numbers and field distributions are presented here for modes of the single-ridge waveguide.

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II. Integral Formulation

The problem is formulated as follows. For waveguides containing only a homogeneous isotropic medium, the electric field within the waveguide is expressed in terms of the vector potential A and scalar potential ϕ as

$$E = -j\omega A - \nabla\phi \tag{1}$$

where

$$A = \mu \oint_C JG(kR)dl \tag{2}$$

$$\phi = \frac{1}{\epsilon}\oint_C \sigma G(kR)dl. \tag{3}$$

Here, $G(kR)$ is the two-dimensional Green's function and can be expressed in terms of $H_0^{(2)}$, the Hankel function of the second kind zero order as

$$G(kR) = \frac{1}{4j} H_0^{(2)}(kR). \tag{4}$$

Also, C is the contour bounding the waveguide cross section, dl is the element of arc along C, R is the distance between a source point and the field point, k is the wave number, and μ and ϵ are the permeability and permittivity of the medium within the waveguide. The quantities J and σ are the wall current and charge, respectively, related by the equation of continuity. The